APPENDIX

STATISTICAL ANALYSIS

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1 Main Statistics of Time Series

For each test run the responses are collected and listed immediately after each run giving the following parameters:

- Mean value of record: $\bar{x} = \frac{1}{N} \sum_{i=1}^{N} X_i$

- Maximum value: $X^{+}_{\max}$ (crest to zero)

- Minimum value: $X^{-}_{\min}$ (trough to zero)

- Standard deviation: $\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} \left(X_i - \bar{x}\right)^2}$

where N is the total number of samples.

X_i is a discrete sample of the parent time series.

2 Peak Value Statistics

MARINTEK's standard statistical program is a computer program which is finding the local, or global peaks (maxima and minima). From one maximum to the following minimum the double amplitudes are also calculated. The values are used to calculate the peak value statistics as specified below. The program can also calculate the cumulative distribution and the statistical density function.

To avoid that very small peaks as for instance signal noise are taken as maxima or minima, some limitations to define the maxima or minima are specified. The standard values are:

- The time step from one extreme value to the next shall be longer than the sample interval.

- The difference between one maximum and the following (previous) minimum shall be more than 2% of the standard deviation σ.

Local peaks: May also include e.g. maxima lower than the mean value.

Global peaks: Defined by mean-crossing.
In our standard post-processing analysis the following parameters are derived:

- Skewness: $\gamma_1 = \frac{m_3}{\sigma^3}$ (expected 0 for a Gaussian process)

- Kurtosis: $\gamma_2 = \frac{m_4}{\sigma^4} - 3$ (also called excess of kurtosis, expected 0 for a Gaussian process)
- Mean value: $\overline{X}$
- Standard deviation: σ
- Number of maximum values: N^+
- Number of minimum values: N^-
- Number of mean value upcrossings: $N^{(u)}$
- Maximum value: $X^+_{\max}$
- Minimum value: $X^-_{\max}$
- Significant double amplitude $(2X)_{1/3}$ i.e. the mean of the one-third highest peak to peak (crest to trough) values (local or global)
- Largest double amplitude $(2X)_{max}$
- Significant maxima; $X_{1/3}^+$ i.e. the mean of the one-third highest crest to zero values (local or global)
- Significant minima; $X_{1/3}^-$ i.e. the mean of the one-third highest trough to zero values (local or global)

For Gaussian single-peaked records, $(2X)_{1/3}$ is normally around 95 - 98 % of 4 σ.

The test results are normally presented in tables.

3 Probability Analysis of Peak Values - Weibull Scale Axis

The cumulative distribution of a randomly selected peak value A_K is written as $P[A_K < A]$, or simply as P[A]. Thus 1-P[A] is the exceedance probability at the given level A. Diagram examples are shown in Fig. D.1. The analysis indicates whether large extreme values are simply results of statistical uncertainties, or results from more systematic trends. Weibull scaled axes are used in the diagrams in order to have the tail of the distributions emphasized. This is achieved by logarithmic axis for A, and the P[A] - axis plotted as 1n [- ln (1-P(A))].

The Weibull distribution.

$$P[A] = 1 - \exp\left[-\frac{1}{G} \left(\frac{A - \overline{X}}{\sigma} \right)^G \right]$$

will then appear as a straight line in the plot.

In the above distribution

$\overline{X}1$ = mean value of record [X_i]
σ = standard deviation of X_i
G = shape parameter describing the slope of the Weibull curve

For G = 2, one has the commonly used Rayleigh model for statistical distribution of peak values, normally assumed valid for linear responses in irregular waves. G = 1 gives the exponential distribution. The Rayleigh curve is indicated with a fully drawn line in the diagrams (based on the actual mean value $\overline{X}1$ and standard deviation σ of the measured record). Examples of sample distributions with G close to 1 and 2, respectively, are shown in Fig. 1.

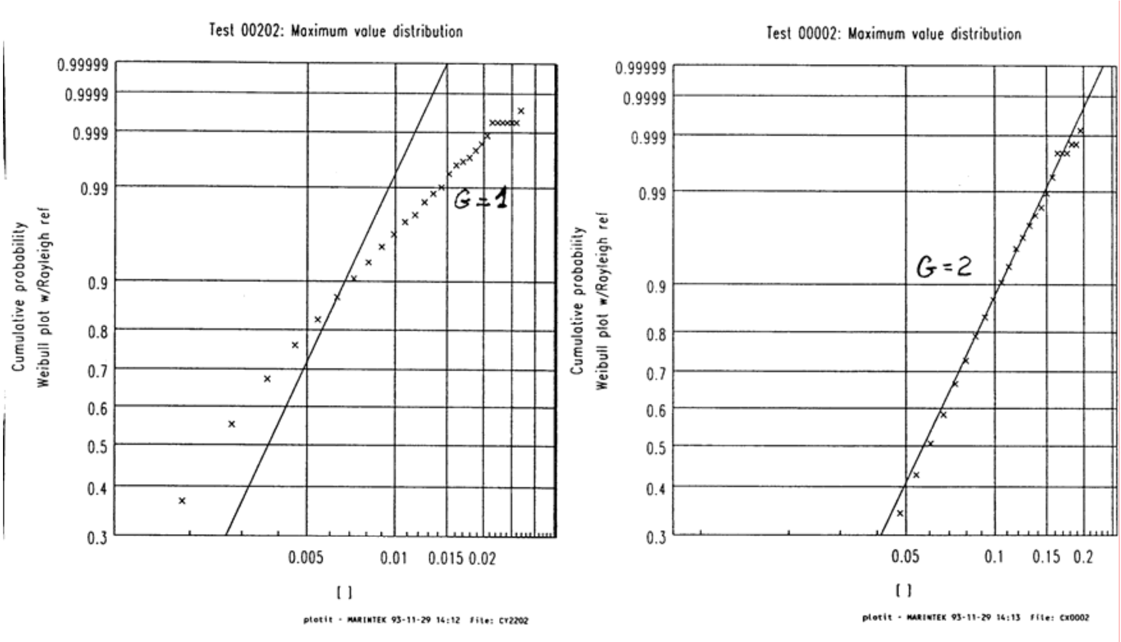

Fig. 1 Example of cumulative probability distributions.