# APPENDIX

# SPECTRAL ANALYSIS

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## 1 Power Spectra

The power spectra are estimated from Fast Fourier Transform (FFT) of the data. To obtain a consistent estimate, the raw spectrum, based on the full length of the time series, is smoothed by a Gaussian-shaped frequency window with a standard deviation $\sigma$. The smoothing bandwidth 2 $\sigma$ is normally 0.02 $H_z$ (model scale), but it depends on the length of the time series.

## 2 Estimates Based on the Power Spectra

The n'th moment of the spectrum is defined by

$$m_n = \int_o^\infty f^n S(f) df$$

The physical significance of some moments are:

$m_o =$ mean square response or area under spectrum or variance $(\sigma^2)$
$m_2 =$ mean square velocity of response
$m_4 =$ mean square acceleration of response

From the moments the following parameters are estimated:

Estimated significant value = $4\sqrt{m_o}$

Energy average period: $T_1 = \frac{m_o}{m_1}, \left( f_1 = \frac{m_1}{m_o} \right)$

Average zero crossing period: $T_2 = \sqrt{\frac{m_o}{m_2}}$

For wave analysis, the spectral peakedness $Q_p$ is also calculated

$$Q_p = \frac{2}{m_o^2} \int_o^\infty f \cdot S^2(f) df$$

In addition the period corresponding to peak value of the spectrum is presented (the spectral peak period).

## 3 Transfer Functions

Linear transfer functions or R.A.O curves and relative phases when required are estimated from cross-spectral analysis, (see the next section) according to the formula:

$$H(f) = \frac{S_{xy}(f)}{S_{xx}(f)}$$

where *H(f)* is the complex transfer function, such that:

RAO = modulus of *H*
Phase = Phase angle of *H*

Here $S_{xy}$ is the cross spectrum between the response y and the reference *x*, and $S_{xx}$ is the reference spectrum (alternatively, the RAO may be defined as the square root of the ratio between the power spectrum of the response divided by the power spectrum of the reference wave).

The R.A.O. curves will be with the following unit:

$$\left[\frac{basic\ unit\ of\ response}{meter}\right]$$

The function is normally plotted in the range where the coherence (see the next section) is larger than 40 %, and where reference spectrum is larger than 1% of its peak value. In seakeeping tests the reference wave is normally corrected to the actual frequency of encounter.

The reference wave used in the calculation of the transfer functions is normally the wave at the location of the model (middle of the Ocean Basin) without the model present.

The results from the spectral analysis are normally presented as shown in Fig. 1. The most important spectral parameters are also presented in tables.

## 4 Coherence Functions and Cross-spectra

The coherence function between 2 signals *x(t)* (reference) and *y(t)* (response) is described as follows:

$$\gamma_{xy}(f) = \frac{S_{xy}(f)}{\sqrt{S_x(f) S_y(f)}} = |\gamma_{xy}(f)| \cdot exkp^{[j\phi_{xy}(f)]}$$

where

$$S_{xy}(f) = \frac{1}{T} < X_T^*(f) Y_T(f) >$$ cross-spectrum between *x* and *y*

where

$$X_T^*(f) \cdot Y_T(f) = |X_T(f)| \cdot |Y_T(f)| \cdot e^{j(\phi_y - \phi_x)}$$

$$S_x(f) = \frac{1}{T} < |X_T(f)|^2 >$$ = autospectrum of *x*

$S_y(f) = \frac{1}{T} < |X_T(f)|^2 >$ = autospectrum of *y*

$X_T(f) = |X_T(f)| \cdot e^{j\phi_x(f)} = \int_{-\infty}^{\infty} dt x_T(t) \exp^{(-j2\pi ft)}$ = Fourier transform of a sample record of length *T* of *x(t)*

$X_T^*(f) = |X_T(f)| \cdot e^{-j\phi_x(f)}$

$Y_T(f) = |Y_T(f)| \cdot e^{j\phi_y(f)} = \int_{-\infty}^{\infty} dt y_T(t) \exp^{(-j2\pi ft)}$ = Fourier transform of a sample record of length *T* of *y(t)*

For $Y_T(f) = H(f) \cdot X_T(f), \gamma_{xy}(f) \equiv 1$

(linear relation between *x* and *y*)

$*$ = complex conjugate

$j$ = imaginar unit

$\phi_x(f)$ = phase of reference

$\phi_y(f)$ = phase of response

$\phi_{xy}(f)$ = relative phase

| | means 'absolute value'

< > means 'statistical expectation value'

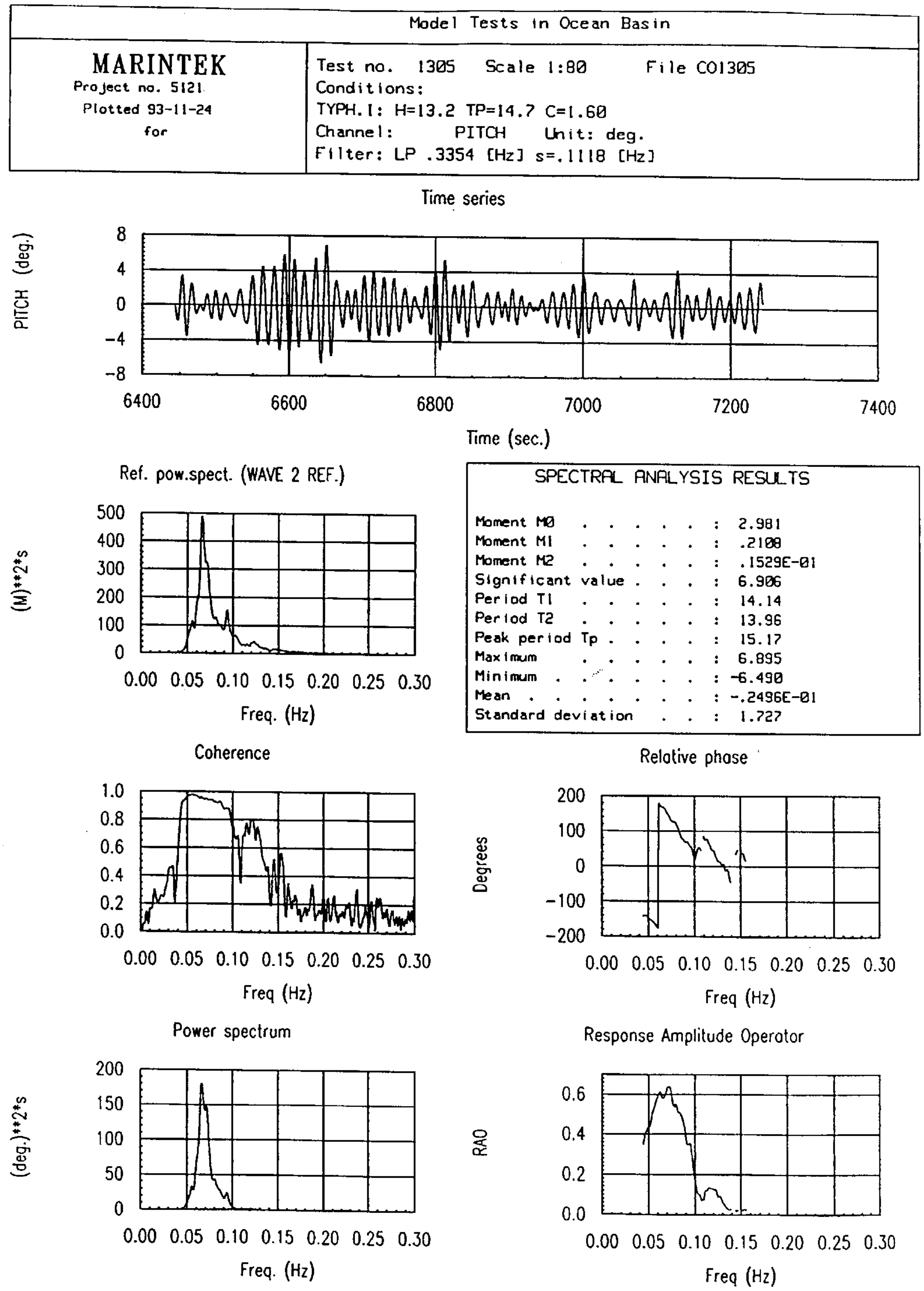


**Fig. 1 Example of results from spectral analysis.**